Brito, L. P., Almeida, L. S. & Osório, A. J. (2021). Seeing in believing: impact of digital simulation pedagogical use in spatial geometry classes. *International Journal of Technology in Teaching and Learning, 17(2),* 109-123.

Seeing is Believing: Impact of Digital Simulation Pedagogical Use in Spatial Geometry Classes

Luciana Pereira de Brito
Polytechnic Institute of Viana do Castelo, Portugal
Leandro Silva Almeida
University of Minho, Portugal
António José Osório
University of Minho, Portugal

Digital simulators have an interesting potential to explain the relationships between two or more variables in a system, facilitating the activation of learning processes and favoring the occurrence of meaningful learning with positive impact on academic performance. To study the impact of the use of digital simulation on learning and academic performance of 9th graders in Spatial Geometry a quasi-experimental study was carried out, with a control group and pre-test/post-test design. The analysis of covariance of the post-test results controlled by the pre-test results revealed a significant improvement in the general academic performance of the experimental group, with a significant improvement of skills related to measurement and associations between nets and solids. Students' positive opinions suggest the experience promoted the occurrence of meaningful learning. Implications for practice point to the pedagogical adoption of such digital resources and special attention to the educational context.

Keywords: meaningful learning; academic success; digital simulation; Spatial Geometry.

INTRODUCTION

The pedagogical mediation that teachers make between the subject to be learned by the student and the cognitive and motivational processes involved in this learning is grounded, to a greater or lesser extent, on didactic resources and related exploration strategies. As meaningfully learning mathematics is probably one of the most demanding cognitive activity in schools (Brito, Almeida, & Osório, 2019), the success of such pedagogical mediation depends on the deliberate, careful, and reflexive articulation of interdisciplinary academic research findings.

Along with sense making, which involves "developing an understanding of a situation, context or concept, connecting it to other knowledge" (NCTM, 2009),

---

Luciana Pereira de Brito is an assistant professor at the Polytechnic Institute of Viana do Castelo and can be reached at lucianabrito@ese.ipvc.pt; Leandro Silva Almeida is a full professor at University of Minho and can be reached at leandro@ie.uminho.pt;António José Osório is an associate professor at University of Minho and can be reached at ajosorio@ie.uminho.pt.
https://doi.org/10.37120/ijttl.2021.17.2.04

mathematical reasoning must occur in all mathematics classrooms (Martin & Kasmer, 2009; Mata-Pereira & Ponte, 2017). However, studies seem to present a different reality. In an exploratory study to identify factors that influence the learning of mathematical concepts by engineering students, Alves et al. (2016) conducted interviews in focus groups with 38 students, of which interpretation and reasoning emerged as the greatest difficulties in learning mathematical concepts. According to one participant in this study, "Mathematics becomes demanding when it stops being just calculus and begins to involve reasoning, since it stops being methodical and mechanical to be more abstract and ambiguous. In secondary school we were used to do similar exercises, and everything that goes beyond that... For most students it becomes difficult to solve problems."

SPATIAL GEOMETRY LEARNING AND PEDAGOGIES

Considering the special case of Spatial Geometry (SG), and just by observing the students' basic need to understand and interpret the two-dimensional views of the three-dimensional objects found in learning resources such as school textbooks, one can understand the urgency of an intentional strategy to also develop the cognitive ability of visualization (Kösa, 2016; Lin & Chen, 2016). In fact, understanding the supreme importance that visual resources have in explaining messages and forming certain concepts, particularly geometrical ones, the study here reported focuses on pedagogical use of digital simulation and its potential to facilitate learning of SG and to improve related academic performance.

While SG may involve every aspect of the study of the space within mathematics, in this study it is narrowed to simple three-dimensional objects like prism, cylinder, pyramid, cone and sphere. As proposed by NCTM (2000) and adapted by Pittalis and Christou (2010), 3D geometric abilities included in Portuguese mathematics curriculum solicit: spatial structuring (e.g. constructing 3D arrays of cubes), whose related activities are mostly developed in early years; conceptualizing mathematical properties in space (e.g. relating the number of vertices, edges and faces of pyramids); measurement (e.g. calculating surface area and volume of 3D objects) and representing 3D objects (e.g. drawing a solid, or recognizing and constructing nets). At the end of middle school, even though the Cavalieri Principle is not to be formally introduced, Portuguese curricular standards prescribe the study of volumes and surface areas of pyramids and cones and the presentation of volume and spherical surface area of spheres.

Brito et al. (2021) found a moderate association between this cognitive ability and Spatial Geometry academic performance, and an explanation of 41% of its variation by sociodemographic, psychological and previous academic performance variables. It is reasonable to argue that a part of the remaining variation will be in charge of contextual variables, which include necessarily the scientific and didactic knowledge of teachers and their practices general pedagogical resources, educational resources and learning experiences. According to de Koning et al. (2002), research has shown the influence of active and meaningful teaching and learning practices to develop students' thinking skills, for example by enriching school activities and modifying teaching methods

DIGITAL SIMULATION TOOLS FOR SG LEARNING AND ITS IMPACT ON LEARNING OUTCOMES

As Falvo (2008, p.75) pointed out more than a decade ago,

> Because development tools are readily available and much easier to use today, the trend of teachers designing and implementing their own animations will continue. As this trend of easy development is combined with broader understanding of how such tools can help teaching and learning, the use of animations and simulations in classrooms will likely grow. If these tools are to live up to their promises to improve teaching and learning in science, researchers must continue to address how to best design and integrate these complex tools into our modern science classrooms.

The pedagogical activity of exploring and visualizing the graphic representations generated by digital simulation is central to this study; we use, first and naturally, the word visualization as the exercise of the sense of sight on these same representations. It is naturally understood that the optical capture of such visual image and basic processing of its shapes, colours and movements results in superior quality perception, organization, understanding and memorization, which will allow, to some extent, to infer more complex aspects of that same image and, desirably, their meanings in the learning contexts in which they are presented. Thus, the pedagogical objective to be achieved by this visualization will be the overall sense making, the awakening of other meanings of this seeing such as watching something, going through, finding, knowing, recognizing, noticing, imagining, inferring, deducting, predicting, proving or calculating. All of these processes are largely associated with the fluid reasoning or student's own thinking, as well as meaningful and progressive learning of the underlying curricular contents.

In digital simulation the user can control variables, formulate hypotheses, interpret information, formulate and try models (Hillmayr et al., 2020; Nafidi et al., 2018), therefore activating inductive and deductive reasoning (Crompton et al., 2018). By providing students with the possibility to manipulate the parameters of a system and thus to actively control the simulation process, the use of simulators promotes a better understanding of the complex and abstract concepts involved, supporting the development of a more critical and reflective thinking (Hillmayr et al., 2020; van der Meij & de Jong., 2006; Yaman et al., 2008). Huang and Chiu (2015) highlight the value of digital simulation in Education, enabling students to actively explore, examine and expand their knowledge by engaging in meaningful learning (Hillmayr et al., 2020). As reported by Hillmayr et al. (2020) in their meta-analysis on the use of digital tools in mathematics and science, "interactive learning environment enables learners to act as sense-makers constructing their own knowledge" (p.2), thus allowing them to develop richer forms thinking about mathematical concepts (Resnick et al., 1998). Renkl & Scheiter (2017) suggest that learning from dynamic and interactive digital images by actively exploring them allows for the regulation of the informational flow to one's level of attention and comprehension. Such features also decrease the spatial visualization cognitive effort required to anticipate the modifications of objects (Crompton et al., 2018; DGE, 2016).

Poon (2018) studied the use of a GeoGebra app to help students compare fractions, finding that not only did students improve their Math skills but also developed positive opinions about using such materials: an increase in motivation and concentration during learning experience, as well as enthusiasm in performing computer tasks. Yang and Yin (2016) investigated in an experiment with a control group and pre and posttests the use of digital simulation of origami in the development of geometric reasoning of 6th grade students, having concluded a positive effect in geometric reasoning and academic performance. The authors suggest further investigations using interviews and observations that allow us to understand the influence of other factors on the success of this strategy.

In his study about the use of robotics, Somyürek (2015) analyzed students' speeches looking for evidence of the occurrence of significant learning. The author concluded that there were constructivist learning experiences observing four distinct categories: active learning, authentic learning, multiple perspectives and collaborative learning. In this experiment, 95% of the students said they had an active role in the classes, this being one of the aspects mentioned as the most important in these classes, as students were not just listening. Students mentioned, "It is more useful because we learn with fun and we produce ideas" and "We have more opportunities to try what we have learned and this helps me to remember easily what I gained" (Somyürek, 2015, p.33). About authentic learning, and focusing especially on improving comprehension, 90% of students pointed in this direction, noting that the course provided a better understanding of previous concepts they had learned in other contexts such as Mathematics or Geography classes. "An activity aiming to move robots down the same road in a mat using different calculations led to a better understanding of units of length such as meters and inches and their conversions" (p.33), and the teachers also mentioned that the students could learn mathematical formulas and their use during the course. About collaborative learning, the authors registered a great majority of positive opinions, noting in particular an opinion that "My friends' thoughts and knowledge that I heard and observed during the activities were helpful to develop my own knowledge and skills about robots and programming" (p.34).

Trundle and Bell (2010) also highlight the success of using simulators – specifically in the acquisition of knowledge – in school performance and in the development of an investigative competence. These authors also highlight the importance of incorporating the deliberate and intentional pedagogical strategy of exploring digital simulators in research, stressing that studies that do not suggest benefits in using simulators did not present the exploration strategy either.

ISSUES OR PROBLEMS

Despite having great potential to foster learning, the use of these materials can also impose an effort on the part of the student (Plass et al., 2009; Renkl & Scheiter, 2017), and the final result does not always correspond to the initial expectations (Renkl & Scheiter, 2017). In fact, and as Renkl and Scheiter (2017) refer, knowledge about these difficulties allows the design of more effective materials to achieve the desired goals. One of the issues raised by these authors is related to student prejudice. Some students seem to prefer to learn from other materials (probably conceiving visual materials as less serious), others are extremely confident in their learning from this type of material, as they perceive an easier learning, resulting in less study time being allocated in future learning situations. Attention to different aspects of the image is also a factor that interferes with learning, as students often focus their attention on irrelevant but visually salient aspects of the image. In the particular case of dynamic images, attention is, therefore, a very important variable, necessary for the desired observation and grasping of changes in position, appearance, orientation and other attributes - "the transience of attributes makes it crucial for the student to be aware of the relevant information at the right time" (Renkl & Scheiter, 2017, p.610). The perception of the usefulness and ease of use of a computer simulator seems to be, according to Liu and Huang (2015), conditioned by the user's level of self-efficacy. According to Bandura's social cognitive theory, self-efficacy is the judgement that a subject has about their own capabilities to organize and execute courses of action that are needed to reach performance goals (Bandura, 1977). At same time, students with greater self-efficacy, they also acquire the level of confidence necessary to operate the simulator.

Finally, and as mentioned by McCrudden and Rapp (2017), to better understand the effectiveness of using this type of resource, some data and methodological variables must be present. It is necessary to know in depth the instructional pedagogy used, such as the engaging and support activities that can improve its use; to focus on the cognitive processes that underlie the attempts to understand the associated visual displays; and finally, to comprehend at least some of the students' cognitive and motivational characteristics and the broader pedagogical contexts in which the simulations are used.

PURPOSE OF THE STUDY

As pointed by Sinclair et al. (2016, p.281),

> The role of technology is just beginning to be understood. At the same time, technology continues to evolve and rapidly change the everyday world and the classroom. Students and teachers are increasingly using digital tools throughout the day and beyond school (Carreira, Jones, Amado, Jacinto, & Nobre, 2016). It is increasingly necessary to understand better how new and emerging digital tools can be used effectively.

Therefore, the purpose of this study was to obtain a deeper understanding about the impact of digital simulation pedagogical use in SG classes on the quality of the learning experience and the related academic performance.

RESEARCH QUESTIONS

According to the purpose of the study, the following research questions are used to guide through the study:

1. Is there a difference on SG academic performance between students who use digital simulation in math classes to learn SG and students who do not use it, when we control for previous SG academic performance?
2. What are students' opinions about the learning experience of SG with digital simulation?

METHOD

DESIGN OF THE STUDY

To answer both questions, a quasi-experimental study was carried out in a natural context, in nine groups (classes) of three Portuguese public schools during 9th grade Spatial Geometry classes that took place in January, roughly in the middle of the school year. The groups were selected not randomly, but for the convenience of geographic proximity, logistics (of having classrooms with computers available for mathematics classes in the experimental group) and also because of the willingness and enthusiasm with the use of technologies in mathematics classes of the teachers participating in the experimental group (EG). This group was formed by four classes (with three teachers, as one of them had two of those classes), and the Control Group (CG) was composed of five classes.

PARTICIPANTS

Participants were 169 students, all 9th graders, distributed almost equally by gender, with 39 girls and 55 boys in the CG and 41 girls and 34 boys in the EG. Ethical standards were applied to solicit their participation.

THE DIGITAL SIMULATION TOOLS

A set of digital simulators – applets – and related activities were designed with GeoGebra software and made available online to support learning in about 13 lessons. Figures 1, 2 and 3 illustrate the most explored simulators during these classes. The S0 simulator (Figure 1) allows interaction only from the perspective of the 3D view, intending to put students in a first contact with the three-dimensional graphic potential of the simulators, facilitating their visualization in space and allowing them to review and consolidate some concepts (namely the characteristics and classification of solids). The simulator S2 (Figure 2) presents the studied solids (cylinder, prism, cone and pyramid) and their nets in a dynamic process controlled by a “wrap”/“unwrap” slider. It is also possible to change the characteristics of each solid. This simulator seeks to explain the mechanical folding process of the planning to obtain the solid, thus intending to reduce the student's cognitive effort and stimulate visualization skills. The association of nets to the corresponding solids was another prior knowledge that was deliberately intended to be retrieved or activated with this simulator.

The S7 simulator (Figure 3) and related activities were designed to support the learning of measurement skills, seeking to highlight similarities and differences between solids, as well as the dimensions of the various elements of the solid and their roles as variables of the various formulas for the various calculations. Several dimensions are presented: the studied solids, their nets, the formulas associated with the calculation of the volume and surface area from their elements and the substitution in the formulas of the different variables, with exact values.

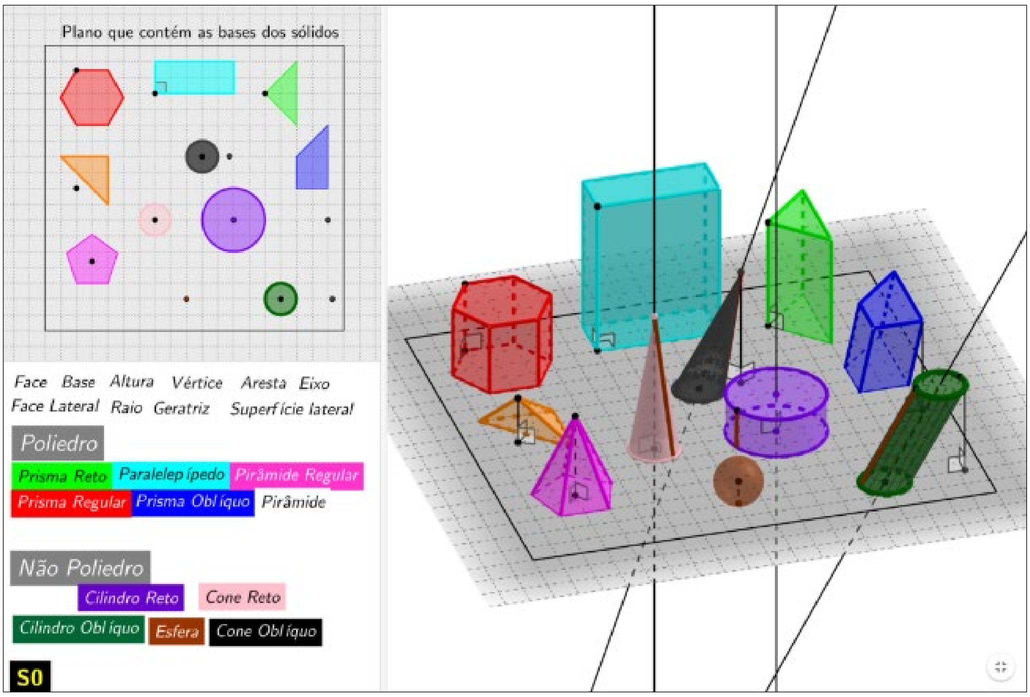
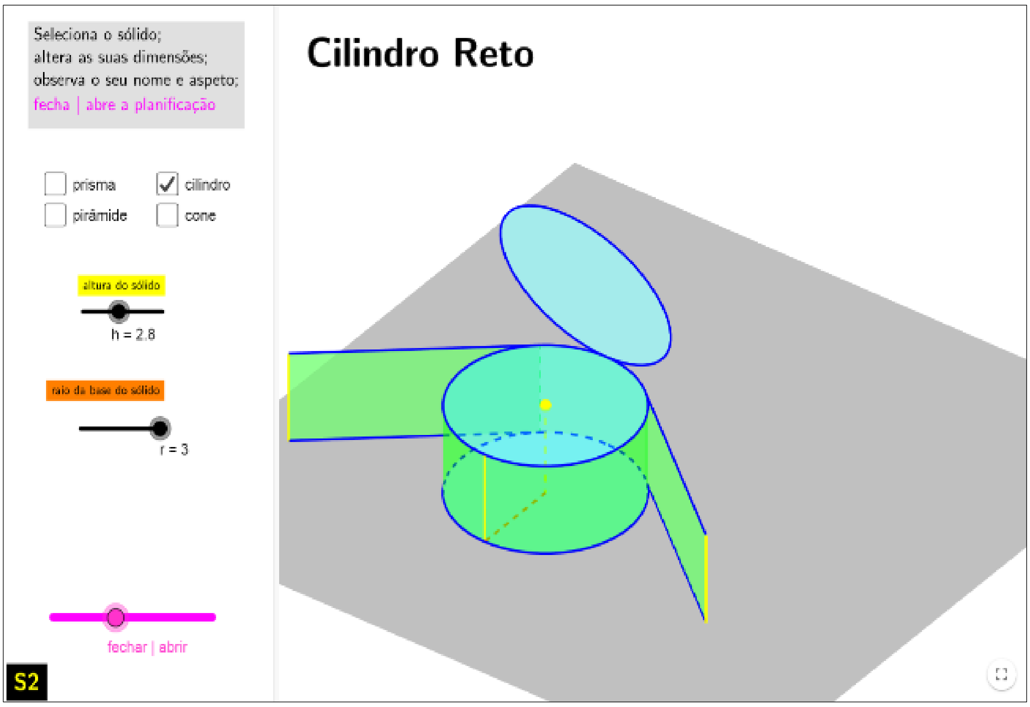


Figure 1. Simulator S0 – revision of some solids.

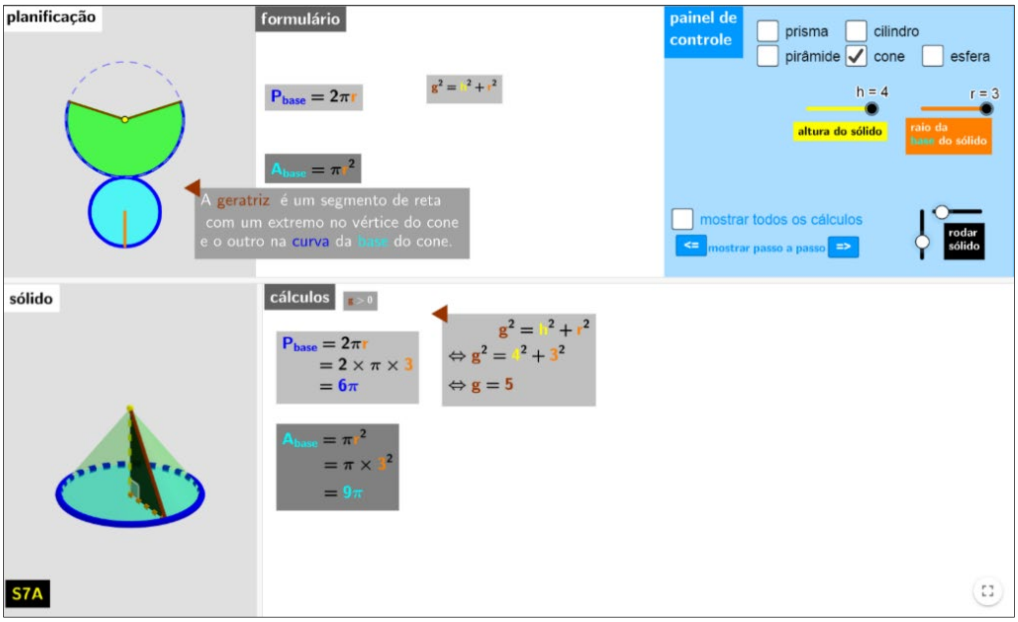

Seleciona o sólido;
altera as suas dimensões;
observa o seu nome e aspeto;
fecha | abre a planificação

☐ prisma ☑ cilindro
☐ pirâmide ☐ cone

altura do sólido
h = 2.8

raio da base do sólido
r = 3

Figure 2. Simulator S2 – solids and their nets



$P_{base} = 2\pi r$

$g^2 = h^2 + r^2$

$A_{base} = \pi r^2$

A geratriz é um segmento de reta com um extremo no vértice do cone e o outro na curva da base do cone.

$P_{base} = 2\pi r = 2 \times \pi \times 3 = 6\pi$

$g^2 = h^2 + r^2 \Leftrightarrow g^2 = 4^2 + 3^2 \Leftrightarrow g = 5$

$A_{base} = \pi r^2 = \pi \times 3^2 = 9\pi$

Figure 3. Simulator 7 – Form, justifications and exact volume and surface area's calculations

The presentation of the form and iterated construction of the intermediate calculations to obtain the volume and surface area could be controlled, with some text notes and right triangles appearing inside the cone/pyramid that justified some of these calculations, as shown in Figure 4.

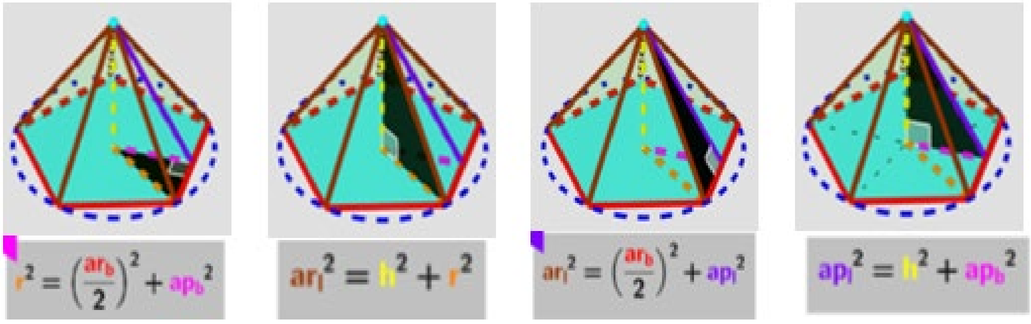


Figure 4. Sequential images of black right triangles within a pyramid, presented in the step-by-step justifications of simulator 7A

PROCEDURES

A didactical plan was conceived to structure the use of the simulators throughout those 13 lessons, which, to avoid threats to the internal validity of the study, was based on the plan that would be followed by the control group. All these materials and the didactical sequence adopted contemplated the activation of prior knowledge, the development of the spiral curriculum, the frequent provision of feedback to students and work in pairs during classes. First, and for nearly four classes, students should recall previous knowledge and skills in SG by exploring simulator S0, S1 and S2 and engaging in related activities. In two classes, the teacher should explore the simulators S3, S4, S5 and S6 and promote the questioning and discussion about geometric principles and formulas for measuring area and volume. In the remaining classes, students should strive to explore simulators 7A and 7B to recall volume and surface area of prisms and cylinders, and evolve this knowledge into pyramids, cones and spheres, and engage in related activities soliciting knowledge application and problem solving.

MEASUREMENTS AND INSTRUMENTS

The pre-test (PR) and post-test (PO) of SG were equal in structure and type of item, with identical items and others very similar, varying only in the fact that students before 9th grade classes had not yet, at least in academic environment, acquired the knowledge and developed skills related to areas of surfaces and volumes of solids such as the pyramid and cone. Therefore, these specific subjects were evaluated only in the post-test and not in the pre-test. Both tests had 20 items and a maximum score of 20 points, with one point for each item.

Spatial Geometry skills evaluated in the tests were:

- C1: conceptualization of mathematical properties in space (identifying number of vertices, edges and faces; items 1a, 1b, 1c;
- C2: representation of 3D objects (recognize and build nets; items 2a, 2b, 2c, 2d, 2e, 2f, 6a, 6b, 6c;
- C3: representation of 3D objects (draw solids; item 4);
- C4: measurement (calculate surface area and volume of 3D objects; items 1d, 3, 5a, 5b, 5c, 7, 8).

From that eight variables were created, four of them regarding students' performance prior to the experiment on each of those SG skills (PR_C1, PR_C2, PR_C3 and PR_C4) and another four regarding the same performances after the experiment (PO_C1, PO_C2, PO_C3 and PO_C4). Another two variables were created, relating to the total scores on PR and PO (PR_total and PO_total, respectively).

The application of these two instruments allowed collecting data to address the first research question, on the quantitative aspect linked to academic performance. Researchers decided to carry out an analysis of covariance of the results of PO_total (dependent variable) controlling with the results of the PR_total (covariate), using group as the independent variable. This technique seemed to be the most appropriate because the addition of a covariate to the analysis model constitutes a statistical error control strategy when this is not possible by resorting to experimental control (Marôco, 2014). By knowing with the pre-test results the initial condition of each student in relation to knowledge and skills in Spatial Geometry – which naturally has a strong influence on the response variable (PO_total) –, the analysis of covariance using PR_total allowed to correct this response variable. To determine the effect size, the value of partial η2 as defined in the SPSS software, and the qualification of the size of this effect as *small, medium,* or *large* was performed as suggested by Cohen (1988).

To complement the data collected and deepen the understanding of the phenomenon of learning with digital simulation, students in the experimental group were asked, at the end of the experiment, to share, in writing, their opinions on the learning experience lived in those classes, which were designed to answer the second research question. Through content analysis, these qualitative data was treated and analyzed, in search for evidence of meaningful learning. The importance given to these contextual and reflective components of the conceived learning experience is emphasized by Trundle and Bell's (2010) concerns about the need, in academic research, to highlight the pedagogical context in which technology is adopted to truly understand its impact on learning and performance.

DATA ANALYSIS AND RESULTS

RESULTS FOR RESEARCH QUESTION ONE

Results presented on Table 1 show the distribution of scores in different variables for pre and posttest by two groups, including skewness and kurtosis coefficients. Observing the results in each group in PR and PO (Table 1), at the global level and in each specific skill of Spatial Geometry, the results obtained express a good variability of the student scores in the two constituted groups. Regarding the mean PR_total score, to be used as a covariate in the ANCOVA to answer the first research question, the difference between the groups was not significant (*t*(133) = 1.95*, p* = .53), suggesting an equivalence between the groups in SG general performance.

Table 1. *Descriptive Statistics of Variables Associated with General Skill in Spatial Geometry and Specific Skills, Before and After the Experiment*

Variable	Group	Min.	Max.	Mean	SD	Skw.	Kurt.
PR_C1	CG	0	3	1.19	.74	.50	.35
	EG	0	3	1.53	.18	-.77	.00
PR_C2	CG	.50	9	5.22	1.68	-.29	.40
	EG	1	9	5.11	.17	-.40	1.00
PR_C3	CG	0	1	.63	.39	-.61	-1.23
	EG	0	1	.70	-.94	-.65	.00
PR_C4	CG	0	5.50	1.55	1.62	.83	-.50
	EG	0	7	2.40	.49	-.83	.00
PR_total	CG	.50	16.00	8.59	3.14	.19	.47
	EG	2	19.25	9.73	4.23	.51	-.56
PO_C1	CG	0	3	1.37	.83	.13	-.48
	EG	0	3	1.95	-.32	-.61	.00
PO_C2	CG	0	8	4.89	1.56	-.41	.42
	EG	0	9	5.40	-.35	.28	.00
PO_C3	CG	0	1	.26	.32	1.14	-.04
	EG	0	1	.45	.21	-1.63	.00
PO_C4	CG	0	5.50	1.52	1.36	.74	.01
	EG	0	7	3.03	-.03	-.63	.00
PO_total	CG	1.25	16.40	8.05	3.02	.27	.22
	EG	1	18.25	10.83	3.50	-.20	-.22

Having verified that the data complied with ANCOVA's assumptions, its results pointed to a significant effect of the pedagogical use of digital simulators in Spatial Geometry academic performance measured by PO_total, after controlling for PR_total, (*F* (1,166)) = 31.31, *p* < .001), with a large effect size (partial η2 = 0.16). After controlling for PR_total, the control group mean score on PO_total was 8.35 with a standard error of .25 and the experimental group mean score was 10.45 with a standard error of .28. Simple contrast analysis revealed that the differences in achievement between learning Spatial Geometry in a pedagogical context with or without digital simulators were highly significant, with Ψ = 2.10, *t* (166) = 5.68 , *p* <.01.

With the ANCOVA we conclude that the use of digital simulators by students working in pairs during about 13 Spatial Geometry classes had a positive impact on their performance in Spatial Geometry, with a large effect size. This result confirms other similar studies, both in Spatial Geometry (Kösa, 2016; Yang & Yin, 2016) and in other areas of Mathematics (Arbain & Shukor, 2015; Diković, 2009; Zengin et al., 2012).

RESULTS FOR RESEARCH QUESTION TWO

Analyzing students' written compositions, where they expressed their opinions on the learning experience, we were able to identify evidence that would allow them to be divided, in general and for each group, into positive or negative opinions, verifying that more than 90% of students considered the experience generally positive. As the students gave their opinion specifically on a) digital simulators, b) work in pairs and, more generally, on c) the experience in Spatial Geometry classes, some categories emerged, such as qualification, feeling or role of the student and the effect on learning, as shown in Table 2.

Table 2. *Content Analysis of Students' Opinions, with Absolute Frequencies and Quotes*

THE LEARNING EXPERIENCE (CLASSES)		
	Positive Opinions	Negative Opinions
Qualification	*Interesting* (7); *fun* (12); *practical* (2); *challenging* (1); *free* (3). “Our teacher often uses GeoGebra to present the contents to us, however, we cannot always explore as freely as we have now”; “in my opinion, I felt freer in these classes, they helped me to better understand the content without being so bound by rules"; better than normal classes (5), “which are boring”; I would like to repeat (4).	-
Feeling and student’s role	*Pleasure* (3); *enthusiasm* (1). “Our teacher's enthusiasm was contagious”; *increased motivation* (7) “Encouraged to learn more and we became more committed”; “All the students, even the ones with the most difficulties, made an effort to try to solve the tasks autonomously”; *increased attention* (1) "I was more attentive because many things called my attention"	I prefer the “normal” classes (5); I had no pleasure (1)
Effects on learning	*Learn/understand better/more easily* (18). “The classes were more effective”; “I learned more because I had never understood this subject”; “it encourages learning, as we see it in practice and not only learn theory”; “we learned, we didn’t “memorized””; “we will never forget how we learn areas and volumes”; “Mathematics made a lot more sense!”; “with the possibility of exploring solids, it was much easier to understand how areas and volumes are calculated. After all, they are not simple formulas!”; “We were able to see the solids and understand how areas and volumes are calculated. When we memorized formulas without understanding them, we easily forgot them. We will never forget, because we were the ones to discover them!” *improved reasoning* (1); *improvement in performance* (4) “I improved my score on the written test, I want more classes like this to improve my grades”.	confusion (2) “I didn't understand so well”, “I was confused”
SIMULATORS		
	Positive Opinions	Negative Opinions
Qualification	*good* (2); *intuitive* (1); *interesting* (2); allowed to *review* the subject (4)	Slow computers (3) Internet failures (2)
Effects on learning	*Facilitate visualization* (18) “I have difficulties with Mathematics and essentially with geometry. Through the use of these materials I discovered a new world. I was able to see it in space”; “I didn't just have to imagine, I could see”; “I was able to see in three dimensions, which until now I had not been able to see”; “I have a lot of difficulties in geometry, because everything seems to me to be very abstract. For the first time I was able to “see” the solids and see how they could relate to each other (…) It is very difficult for students to understand how the areas and volumes of solids are calculated because they cannot visualize these solids. So it was much easier!”; “when rotating the solids one can see better how they are”; "I could see better the planning and the solid."	-

Effects on learning	*Make the formulas easier to understand* (5) “We deduced the formulas and from there they gained meaning, they are no longer an ordered set of words with some numbers that existed on the last page of the manual”; “Formulas have gained meaning: they are no longer a set of memorized letters.” *Interactivity* (being able to move/change/choose) (9) “we can change the values of different variables and see their effects on the solid, on its area and on its volume. All this was impossible without these resources”; “the features presented were very dynamic and allowed movement, which made the study of geometry more real and less abstract and therefore easier for students”; “when rotating the solids one can see better how they are”; "I could feed my curiosity." *Regulation* with exercises and feedback (4) “doing exercises at the same time is good”; "giving the answer right away helps us to understand if we are understanding the subject."	-
	WORKING IN PAIRS	
	Positive Opinions	Negative Opinions
Qualification	*good* (4); *fun* (4); *relaxed* (2)	-
Feeling and student’s role	-	I hardly talk to the colleague I worked with (1) I didn't like it (1)
Effects on learning	*To help* each other and *ask questions* (9);to *share* ideas (1); better learning (2)	*less productive* (1) "Individual work would have been more productive"

By analyzing students’ responses, we conclude that the use of simulators in classrooms by students themselves solicited them a more active role (Nafidi et al., 2018). In addition, due to the interactive nature of the digital resources, we believe that their manipulation and exploration may have contributed to a) the regulation of the information flow at student’s level of attention and understanding (Renkl & Scheiter, 2017), b) reduce the load of working memory (Renkl & Scheiter, 2017), c) decrease the cognitive effort of spatial visualization of anticipating object modification (Crompton et al., 2018; DGE, 2016), and also d) activate inductive and deductive reasoning (Crompton et al., 2018), those latter necessary, respectively, to the formation of conjectures and generalizations and to the deduction of correct answers to the proposed activities. Not directly associated with the use of simulators, but with the more global context of learning that was designed with the didactic planning, we can also assume that students may have benefited from working in pairs (Lourenço & Machado, 2017) and receiving automatic feedback at the end of many activities, this latter contributing to increasing students’ confidence in their own skills (Peixoto et al., 2017).

Simulators must, of course, be designed to develop the desired skills, explaining the relationships that are intended to be communicated and promoting the learning that is intended to be carried out. It is up to the teacher to carry out the appropriate questioning to activate the cognitive processes of induction and deduction in the students, which will occur in the cadence and with the repetition that the student, by manipulating the simulator, wants or needs. In this sense, and regarding the usability of the simulators, the perception of effectiveness and ease-to-use students manifested probably contributed to an increase of motivation (Falvo, 2008).

At the same time, we also consider that, by stimulating student’s interest, this characteristic of interactivity and control of the simulation self-feeds the pro-activity that is necessary for the occurrence of meaningful learning, transforming the effort required by the student to play a more active role in a naturally more involved participation, given the characteristics of the resources to be explored. These conclusions are also based on the complement to the quantitative data that was offered by the students' reports, which also confirmed a general feeling of satisfaction registered by other authors and which was responsible for an improvement in the quality of the learning environment.

CONCLUSION

Meaningful learning is based on understanding and involves students’ cognitive, motivational and behavioral processes, all of which can, in fact, be facilitated by active teaching methodologies and facilitating learning contexts, such as, and as found with this study, the ones involving digital simulation technology. The sense of self-efficacy among teachers is also increased by observing the manifestations of understanding of concepts and procedures by different students, as well as by the positive impact on academic performance. This latter will, therefore, be a motivating factor for students and teachers, promoting in both a sense of self-efficacy and functioning as an unavoidable asset in school culture, which itself can promote the adoption by teachers of this type of digital educational resource and associated pedagogical activities. Naturally, the effort to change is not just the responsibility of teachers and students. Schools are the context where they all interact and consolidate their beliefs about teaching and learning. This fact transforms the problem of change into an issue that is rooted in the context: “it is crucial to adopt an ecological model, in which innovation can be ensured not only in the agency of teachers, but which can also involve school and institutional leaders of more macro order” (Freires et al., 2019, p. 765). It is necessary to break down the barriers that prevent the adoption of technology - such as the lack of time, access, resources, knowledge and support (Raja & Nagasubramani, 2018) - equipping schools with computer material and pedagogical and collaboration spaces, which allow the use of technology and widespread access to the internet. In fact, it is easily assumed that a teacher's level of enthusiasm for the use of technology must be very high to implement such use in a school without computers or multimedia projector or where requesting a room with computers is a time-consuming process. It is also necessary to create in schools teams that unconditionally support the use of technology for learning, that support enthusiastic teachers in their self-training processes, encourage them to share these practices with peers and that present real benefits of using technology to improve the quality of learning and the academic performance.

LIMITATIONS AND FURTHER STUDIES

One of the main limitations of this research is that the conclusions cannot be reliably generalized, as schools and classes were not randomly selected. Furthermore, and to address the concerns of Trundle and Bell (2010) and Hillmayr et al. (2020), conclusions about the impact of simulators should consider the specific set of simulators designed for this study, and the pedagogical approach adopted in the didactic planning, which included a set of activities carried out mainly in pairs only by the experimental group, with many activities with self-correction and immediate feedback.

Further studies could focus on many different aspects. First, the observation of contexts or conditions not studied in this investigation. Propose to the control group the same exercises/activities and learning context, which would certainly lead to a more refined result on the impact of digital simulation pedagogical use. Although the teachers in the experimental group shared with the students the address of the website where the simulators were located, there was no control over the frequency or duration of students’ access to the simulators outside the classroom. Have students accessed the simulators outside of classes, willingly, with curiosity? Was it boys and/or girls? Was there a relation between the outside school accesses and the improvement in SG academic performance? Also observing that students worked in pairs, we could not verify if there was a true connection/collaboration during the activities, or if one of the students was always manipulating the simulators and the other just passively observing. Would the positive impact of the use of digital simulation have produced an even greater effect size if each student could have manipulated the simulators, while freely consulting their colleagues, in an environment of collective knowledge construction? The exploration time may possibly have a relationship with the effect that the visualization of the simulators' products has on the student's cognitive structure. Secondly, it seems important to carry out studies with the same methodological design but rooted in other themes, whether from Mathematics or from other areas of knowledge.

Considering that the results of such studies will only have an impact on practice if teachers choose to incorporate these tools in their classes, and considering that the teacher is the great mediator between the content to be learned and the student learner, studies on learning with digital simulation in a natural context would certainly be enriched with the assessment of dimensions linked to the teacher, such as those studied in the Technology Acceptance Model (TAM) or the Technological Pedagogical Content Knowledge (TPACK).

Regarding teacher training to promote the pedagogical adoption of digital simulation in their classes, we raise the question: knowing that, from training, teachers only take what fits their beliefs (Gilakjani, 2012; Stipek et al., 2001), could the very design of digital simulators, which is based on a comprehensive knowledge of the scientific principles they intend to communicate, revolutionize teacher training in the technical, scientific and pedagogical components? In the particular case of Mathematics and GeoGebra programming, such processes involve knowledge of Logic, Numbers, Functions, Analytical Geometry and basic programming principles, among other domains, depending on the complexity of what one intends to build. As suggested by Rubio et al. (2016), the design of digital simulation will be by itself and for teachers an experimental learning experience, where the application of many of the concepts and procedures they teach in their professional practice is required, allowing them to contextualize their own learning and attribute new meanings and purposes to these same mathematical elements. In the case of designing simulation for other areas of knowledge, the call for collaborative work and articulation of knowledge in interdisciplinary projects will certainly emerge, a growing reality in educational and training contexts.

REFERENCES

Alves, M., Coutinho, C., Rocha, A., & Rodrigues, C. (2016). Fatores que influenciam a aprendizagem de conceitos matemáticos em cursos de engenharia: Um estudo exploratório com alunos da Universidade do Minho. *Revista Portuguesa de Educação, 29*(1), 259-293.

Arbain, N., & Shukor, N. A. (2015). The Effects of GeoGebra on Students Achievement. *Procedia - Social and Behavioral Sciences, 172*, 208-214.

Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review, 84*(2), 191–215.

Brito, L. P., Almeida, L. S., & Osório, A. J. M. (2020). Reasoning Abilities and Learning Math: A Möbius Strip?. International Electronic *Journal of Mathematics Education, 15*(2), em0565.

Brito, L.P., Almeida, L., & Osório, A. (2021). Academic performance of 9th graders on Spatial Geometry: Impact of personal and contextual variables. *BOLEMA, 35*(70), 690-707.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. Second Edition. Lawrence Erlbaum Associates, Publishers.

Crompton, H., Grant, M., & Shraim, K. (2018). Technologies to Enhance and Extend Children’s Understanding of Geometry: A Configurative Thematic Synthesis of the Literature. *Journal of Educational Technology & Society, 21*(1), 59-69.

de Koning, E., Hamers, J. H. M., Sijtsma, K., & Vermeer, A. (2002). Teaching Inductive Reasoning in Primary Education. *Developmental Review, 22*(2), 211-241.

DGE, (2016). Orientações de gestão curricular para o Programa e Metas Curriculares de Matemática do Ensino Básico. Ministério da Educação.

Diković, L. (2009). Applications GeoGebra into Teaching Some Topics of Mathematics at the College Level. *Computer Science and Information Systems*(12), 191-203.

Falvo, D. (2008). Animations and simulations for teaching and learning molecular chemistry. *International Journal of Technology in Teaching and Learning, 4*(1), 68–77.

Freires, T., Bento, M., Morgado, J. C. & Lencastre, J. A. (2019). Professores veteranos e inovação curricular: desafios do recurso à tecnologia como instrumento pedagógico. II Seminário Internacional CAFTe - Currículo, Avaliação, Formação e Tecnologias Educativas (pp. 761-772). Porto: Faculdade de Psicologia e de Ciências da Educação da Universidade do Porto.

Gilakjani, A. P. (2012). The relationship between teacher pedagogical beliefs and use of computer technology in the classroom. *International Journal of Information and Education Technology, 2*(1), 11-18.

Kösa, T. (2016). The effect of using dynamic mathematics software: cross section and visualization. *The International Journal for Technology in Mathematics Education, 23*(4), 121-128.

Lin, C.-H., & Chen, C.-M. (2016). Developing spatial visualization and mental rotation with a digital puzzle game at primary school level. *Computers in Human Behavior, 57*, 23-30.

Liu, C. H., & Huang, Y. M. (2015). An empirical investigation of computer simulation technology acceptance to explore the factors that affect user intention. *Universal Access in the Information Society, 14*(3), 449-457.

Lourenço, R., & Machado, J. (2017). Aprender juntos: projeto de apoio curricular entre pares. *Revista Portuguesa de Investigação Educacional, 17,* 124-143.

Martin, W., & Kasmer, L. (2009). Focus in High School Mathematics: Reasoning and Sense Making (Executive Summary). NCTM.

Marôco, J. (2014). *Análise Estatística com o SPSS* (6^a edição). ReportNumber.

Mata-Pereira, J., & Ponte, J.-P. (2017). Enhancing students' mathematical reasoning in the classroom: teacher actions facilitating generalization and justification. *Educational Studies in Mathematics, 96*(2), 169-186.

McCrudden, M. T., & Rapp, D. N. (2017). How Visual Displays Affect Cognitive Processing. *Educational Psychology Review, 29*(3), 623-639.

Nafidi, Y., Alami, A., Zaki, M., El Batri, B., & Afkar, H. (2018). Impacts of the use of a digital simulation in learning earth sciences (the case of relative dating in high school). *Journal of Turkish Science Education (TUSED), 15*(1), 89-108.

NCTM (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

NCTM (2009). Focus in High School Mathematics: Reasoning and Sense Making. The National Council of Teachers of Mathematics Inc.

Peixoto, F., Sanches, C., Mata, L., & Monteiro, V. (2017). "How do you feel about math?": Relationships between competence and value appraisals, achievement emotions and academic achievement. *European Journal of Psychology of Education, 32*(3), 385-405.

Plass, J. L., Homer, B. D., & Hayward, E. O. (2009). Design factors for educationally effective animations and simulations. *Journal of Computing in Higher Education, 21*(1), 31-61.

Poon, K. K. (2018). Learning fraction comparison by using a dynamic mathematics software – GeoGebra. *International Journal of Mathematical Education in Science and Technology, 49*(3), 469-479.

Raja, R., & Nagasubramani, P. C. (2018). Impact of modern technology in education. *Journal of Applied and Advanced Research, 3*(1), 33-35.

Renkl, A., & Scheiter, K. (2017). Studying visual displays: How to instructionally support learning. *Educational Psychology Review, 29*(3), 599-621.

Rubio, L., Prieto, J., & Ortiz, J. (2016). La matemática en la simulación con GeoGebra. Una experiencia con el movimiento en caída libre. IJERI: *International Journal of Educational Research and Innovation*, (5), 90-111.

Somyürek, S. (2015). An effective educational tool: Construction kits for fun and meaningful learning. *International Journal of Technology and Design Education, 25*, 25-41.

Sinclair, N., Bartolini Bussi, M. G., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2016). Recent research on geometry education: an ICME-13 survey team report. *ZDM, 48*(5), 691-719.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213-226.

Trundle, K., & Bell, R. (2010). The use of a computer simulation to promote conceptual change: A quasi-experimental study. *Computers & Education, 54*(4), 1078-1088.

Yang, H. H., & Yin, S. K. (2016). A study of elementary school students' geometric reasoning using digital origami simulation tool. *The International Journal of E-Learning and Educational Technologies in The Digital Media, 2*(1), 1-8.

Zengin, Y., Furkan, H., & Kutluca, T. (2012). The effect of dynamic mathematics software geogebra on student achievement in teaching of trigonometry. *Procedia - Social and Behavioral Sciences31*, 183-187.